7.4 Area of a surface

We learn:

- The formula that will give the area of a surface, given a parametrization of the surface.
- Why it works.

Review. When we did integrals along curves there were two kinds:

- an integral giving the length of the curve, or giving the mass of a wire from its line density
- an integral giving the work done by a vector field in moving along the path.

We assume we have a parametrization
Phi : D -> R^3 $D \subseteq \mathbb{R}^2$
satisfying conditions: Phi is 1 - 1, differentiable with continuous partial derivatives, and regular.

The formula:

$$\text{Area} = \iint_D \| T_u \times T_v \| \, du\, dv$$

Example: the unit sphere:
Phi(u,v) = (sin v cos u, sin v sin u, cos v)

T_u = (- sin v sin u, sin v cos u, 0)
T_v = (cos v cos u, cos v sin u, -sin v)

T_u x T_v = (-sin^2 v cos u, -sin^2 v sin u, -sin v cos v)

|| T_u x T_v || = $\sqrt{\text{sin^4 v + sin^2 v cos^2 v}}$ = |sin v|

Area = $\int_0^{2\pi} \int_0^{\pi} \sin v \, dv \, du \approx 4\pi$

Note $|\sin v| = \sin v$ when $0 \leq v \leq \pi$.

Why it works: